CAMBRIDGE

Sixth Term Examination Paper [STEP]

Mathematics 2 [9470]

2024

Examiners' Report

Mark Scheme

STEP MATHEMATICS 2

2024

Examiners' Report

STEP 2 Introduction

Many candidates produced good solutions to the questions, with the majority of candidates opting to focus on the pure questions of the paper. Candidates demonstrated very good ability, particularly in the area of manipulating algebra. Many candidates produced clear diagrams which in many cases meant that they were more successful in their attempts at their questions than those who did not do so. The paper also contained a number of places where the answer to be reached was given in the question. In such cases, candidates must be careful to ensure that they provide sufficient evidence of the method used to reach the result in order to gain full credit.

Question 1

There were a large number of attempts at this question, with many good answers seen and many attempting most parts of the question.

Many were able to show that the required formula in part (i) will be satisfied if the stated sum is satisfied, but many did not explain that the result applies in both directions with sufficient detail. Of those who failed to show the result in this part the main error was an incorrect choice of limits when expressing the sums either in sigma notation or as an arithmetic series.

Candidates generally demonstrated an understanding of what was required for part (ii), but a significant number did not express their solution in a clear form, for example by finding just one case or listing the first few without specifying a general relationship. A number of candidates got confused between squares and square roots and having deduced that the square root of n is equal to c then concluded that n must be a square number.

In part (iii) most candidates successfully identified the equation that needed to be satisfied, but were unable to explain clearly why there were no solutions.

Part (iv)(a) was generally completed well, including by candidates who had struggled in earlier parts of the question. Many good answers to part (iv)(b) were also seen, although a number of cases did not explicitly identify the relationship that must exist between the values of N and K.

Most candidates recognised that the result of part (iv)(b) could be used to generate the further solutions required in part (iv)(c).

Question 2

This question was attempted by approximately three-quarters of the candidates and many very good solutions were seen.

In part (i) candidates were generally able to find the binomial series expansion, but some did not give a clear enough statement of the general term. A number of candidates did not recognize that it would be possible to use the series expansion to establish the integration result that was required and instead attempted to use integration by parts or to produce a proof by induction.

Most candidates recognized the way in which part (i) could be applied to answer part (ii) and this part was generally answered well. A number of candidates forgot to evaluate the integral before moving on to the next part of the question.

In part (iii) a large number of candidates recognized that the use of partial fractions would allow them to apply part (i) again, but many did not then realise that there was a need for partial fractions to be applied a second time. Almost all candidates who reached the final integral were able to recognize it and either state the answer or use an appropriate substitution.

Question 3

This question was attempted by approximately three-quarters of the candidates, but only a few were able to achieve a fully complete solution to the question. This question was one where a diagram was very helpful and approaches that were supported by geometrical understanding were generally more successful than attempts that relied solely on algebra.

Part (i) was very well done, but candidates who used geometrical arguments generally did not address cases not covered by their diagram – usually this was the case where the value of θ was negative.

Part (ii) was also done well, but some candidates failed to give enough working to support their answer in (a), which is very important in questions where the answer is given. Similarly, in (b) a number of candidates did not show clearly how they interpreted their algebraic work to reach a geometrical description.

Part (iii) was found to be difficult by a large number of candidates. Part (a) was generally done well, although some care with the algebra and exact trigonometric values was needed. Many were then unable to identify a relationship between f_3 and the previously seen functions and did not reach a correct geometrical description in words. A small number did well on part (c) and were able to interpret the inverse of function f_2 geometrically, but very few reached a fully simplified geometrical transformation.

Question 4

About one third of candidates attempted this question. Many of the attempts struggled to explain the reasoning sufficiently clearly, often missing one or two important details.

In part (i) most candidates were able to prove that the cosines of the two angles were equal, but did not justify details such as checking that everything lies in the same plane. The most problematic part of this part of the question for candidates was (i)(b). The most successful approach was to use the fact that *XB* and *BY* lie on the same line and then use algebra to calculate the value of λ and the ratio. A small number of candidates were able to achieve full marks with geometric arguments, but most such attempts did not give sufficient justification to be fully convincing. Candidates often produced successful attempts at (i)(c), although again some failed to justify elements of the solution fully enough in some parts.

Part (ii) was found to be harder than the rest of the question. The majority who attempted this part correctly identified the dot product to be considered, but most did not recognize the symmetry within the three expressions.

Question 5

This was the second most popular question, but many candidates did not recognise the significance of the domain being used for the functions in this question. This meant that many applied techniques relevant to functions where the domain is the set of real numbers and therefore reached incorrect answers.

The first requirement of part (iv) was not affected by this misunderstanding and most candidates were able to prove the required result successfully, usually by completing the square.

A number of good solutions were seen, however. Those who completed the square and used the difference of two squares were often successful in parts (i), (ii) and (iii). Some of these were also able to make progress on the end of part (iv).

Question 6

A large number of attempts at this question were seen and many candidates were able to produce good solutions.

Part (i) was answered well, although a number of candidates did not select the correct base case. Several solutions did not give sufficient detail in the proof to gain full credit however. Since the required form is known, it is important that steps in the solution are shown clearly.

In part (ii) many candidates started by calculating some of the terms and then attempted to spot a pattern, or match the terms to the desired result. To gain full credit a solution that showed the form of the general term of the binomial series expansion was required. In a small number of cases combinatorial coefficients with non-integer arguments were used, but no explanation was given of the meaning of this notation.

In part (iii) a number of candidates again did not write down the binomial coefficients and used pattern spotting. In general, candidates who had successfully answered part (ii) were also successful in part (iii).

Some good solutions to part (iv) were seen, but often there was insufficient detail in the solutions to gain full credit.

Question 7

While there were many attempts at this question, most did not achieve very high marks.

The sketch of the graph in part (i) needed to be clear that it was two circles each with a radius of 1, for example by stating that points on the curve must satisfy the equation of one of the circles, or by marking the centres of the circles and having the correct radius in each case.

Most candidates were able to reach the required result at the start of part (ii)(a). Candidates often missed important features of the analysis in the remainder of this part, usually believing that the conclusions derived from considering the discriminant of the quadratic in x^2 was sufficient to analyse the number of roots in each case.

In part (ii)(b) candidates were able to use their results from (ii)(a) to find the maximum value for y, but many struggled to obtain the correct value for x or incorrectly considered distances from the x-axis rather than from the y-axis. In

(ii)(c) candidates often struggled to explain their reasoning clearly enough to give a fully convincing answer. In general candidates were comfortable in using algebra to prove that the two brackets cannot be negative at the same time and many were able to explain why this would mean that they must both be positive, but found it difficult to explain the significance of this in terms of the graph.

In part (ii)(d) candidates often produced a graph that contained some of the features that had been deduced earlier in the question. Many solutions did not show intersections with axes or co-ordinates of maximum and minimum points.

Question 8

This question was attempted by approximately half of the candidates.

In part (i) candidates were often able to produce the necessary algebra, but some did not justify the strictness of the inequality or failed to use a full inductive structure for the proof. There was a roughly even split between candidates who identified that x_0 could be used as a bound in order to apply the given result and those who incorrectly used the fact that $y_n < x_n$. When considering the behaviour of $(x_n - y_n)$ some candidates incorrectly asserted that the fact that the sequence is bounded below by 0 is sufficient to show that the sequence tends to 0. Almost all candidates were able to show that this result implies that the two sequences tend to the same limit.

In part (ii) most candidates were able to apply the substitution, but some did not justify the new values of the limits or comment on the evenness of the integrand. The evaluation of the final integral was generally done well.

Question 9

About one-third of the candidates attempted this question. In some cases the diagrams that were drawn showed that candidates had not taken care to fully understand the description of the situation.

Part (i)(a) was completed well by many candidates, but some did not recognise the need to use a trigonometric identity to obtain a form which is a function of $\tan\alpha$ which is a common technique in questions of this form.

Part (i)(b) was mostly done well, although most did not score fully on the final deduction, often not applying the condition that $\lambda > 1$ correctly and reaching two cases, one of which they could not rule out.

Very few candidates produced a good solution to part (ii), but those who managed to identify the correct starting point generally produced good solutions.

Question 10

This question received the fewest attempts, although a good proportion of the attempts that were made were successful. Many candidates drew clear diagrams in their attempts at this question.

Part (i) was done fairly well, with most candidates resolving forces successfully. However, many candidates were not able to justify sufficiently well the situation in equilibrium as opposed to limiting equilibrium.

In part (ii) many candidates struggled to know how to deal with the force that acts on the prism and thought instead that it would be acting on the particle.

Those candidates who attempted part (iii) generally did well and many realised how to get both sides of the required inequality and were able to follow through the required manipulation.

Question 11

This question received one of the lowest numbers of responses and many of the responses did not achieve many marks.

In part (i) candidates were generally able to complete the differentiation correctly and identify the location of the stationary point of the curve. Most were also able to identify the correct behaviour of the course as $x \to 0$, but several incorrectly believed that the function also approached 0 as $x \to \infty$. Some candidates were not able to justify why the maximum value when taking the function over integers must occur when $n = 2$ or $n = 3$. Many candidates were however able to explain clearly that the value is greater when $n = 3$ compared to $n = 2$.

Part (ii) proved to be difficult for many candidates, with many incorrectly calculating the probability that a combined test is found to be negative or omitting the first test when counting the number of tests required if a group did test positive.

Those candidates who had successfully solved part (ii) were able to produce good solutions to part (iii) as well. Part (iv) was also answered well, although several candidates did not justify the exclusion of higher order terms in the expansion of $(1 - p)^k$.

Question 12

This question received one of the lowest numbers of responses and many of the responses did not achieve many marks.

In both parts of the question candidates often failed to calculate the probabilities that were needed to start the calculations, although many candidates were able to calculate the relevant combinatorial factors successfully and showed accurate algebraic manipulation when using the formulae provided in the question.

Those candidates who reached the final part of the question were able to identify a suitable approach for comparing the probability of there being one winner with the probability of there being two winners, although this was not executed correctly in most cases.

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